An ILP-based improvement method for the travelling salesman problem

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14th June 2018

The *travelling salesman problem* (*TSP*) is one of the most prominent $\mathcal{NP}$-hard combinatorial optimization problems. Given a complete graph $G = (V, E)$ and non-negative distances d for every edge, the TSP asks for a shortest cycle through all vertices with respect to the distances d.

Exact algorithms are usually based on integer linear programming (ILP), often utilizing a branch-and-bound or branch-and-cut approach. Heuristically considered, besides classical construction heuristics as the nearest neighbour (NN) or farthest insertion (FI) and improvement methods like the famous k-opt or Lin-Kernighan (LK) procedures, also metaheuristics play a meaningful role. Recently, with the rise of commercial solvers, which is supported by the growing availability of more capable hardware, incomplete or partially exact linear optimization methods, so-called *matheuristics*, have been established. These models rely on ILPs and/or their relaxed versions, i.e. MILPs or (standard, real-valued) LPs. We propose such a matheuristic, an improvement procedure for the solving of the (symmetric) Euclidean TSP, where the vertices correspond to points in an Euclidean plane.

Similarly to *large neighbourhood search* methods, our *magnifying glass matheuristic* (MGM) partially destroys an initial solution by removing sub-paths in a specific area, given by the current position of a "magnifying glass" in the graph, and then repairs it optimally by means of an ILP. This is iteratively repeated until all sections of the graph are examined, while usually improving (but never worsening) the overall tour at various places. This approach was recently successfully introduced for the so-called *quadratic travelling salesman problem* (QTSP) in [1].

Using base solutions from NN and FI, variants of MGM were tested on randomly generated instances based on uniformly distributed points in the Euclidean plane and on a set of DIMACS instances with sizes ranging up to several hundred thousand vertices. All results, put into relation to those of 2- and 3-opt heuristics and to LK, illustrate that

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MGM significantly outperforms the k-opt approaches in both the objective function values and the running times and that it constitutes a competitive variant to the more sophisticated procedures like LK.

Keywords. Travelling salesman problem; integer linear programming; matheuristic; improvement heuristic

References

[1] R. Staněk, P. Greistorfer, K. Ladner, and U. Pferschy, Geometric and LP-based heuristics for the quadratic travelling salesman problem, submitted to *Computers & Operations Research*, available as `arXiv:1803.03681`.