Solution methods for the quadratic TSP and its variations

Rostislav Staněk[*]   Oswin Aichholzer[†]   Anja Fischer[‡]

Frank Fischer[§]   J. Fabian Meier[¶]   Ulrich Pferschy[‖]

Alexander Pilz[**]

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The *traveling salesman problem* (*TSP*) asks for a shortest tour through all vertices of a graph with respect to the weights of the edges. The *symmetric quadratic traveling salesman problem* (*SQTSP*) associates a weight with every three vertices traversed in succession. If these weights correspond to the turning angles of the tour, we speak of the *angular-metric traveling salesman problem* (*Angle TSP*).

First, we consider the SQTSP from a computational point of view. In particular, we apply a rather basic algorithmic idea and perform the separation of the classical subtour elimination constraints on *integral solutions* only. It turns out that this approach is faster than the standard fractional separation procedure known from the literature. We also test the combination with strengthened subtour elimination constraints introduced by Fischer and Helmberg in 2013.

Secondly, we provide a different, mathematically interesting *MILP linearization* for the Angle TSP that needs only a linear number of additional variables while the standard linearization requires a cubic one.

Finally, we introduce *MaxSQTSP*, the maximization version of the quadratic traveling salesman problem. Here it turns out that using some of the *stronger subtour elimination constraints* helps. For the special case of the *MaxAngle TSP* we can observe an interesting geometric property if the number of vertices is odd. We show that the

[*]`rostislav.stanek@uni-graz.at`. Department of Production and Operations Management, University of Graz, Universitätsstraße 15/E3, A-8010 Graz, Austria

[†]`oaich@ist.tugraz.at`. Institute for Software Technology, Graz University of Technology, Inffeldgasse 16B/II, A-8010 Graz, Austria

[‡]`anja.fischer@mathematik.uni-goettingen.de`. Institute for Numerical and Applied Mathematics, University of Göttingen, Lotzestraße 16–18, D-37083 Göttingen, Germany

[§]`frank.fischer@uni-kassel.de`. Institute for Mathematics, University of Kassel, Heinrich-Plett-Straße 40, D-34132 Kassel, Germany

[¶]`brief@fabianmeier.de`. Continentale Krankenversicherung a.G., Ruhrallee 96, D-44139 Dortmund, Germany

[‖]`pferschy@uni-graz.at`. Department of Statistics and Operations Research, University of Graz, Universitätsstraße 15/E3, A-8010 Graz, Austria

[**]`alexander.pilz@inf.ethz.ch`. Institute of Theoretical Computer Science, ETH Zurich, Universitätßtrasse 6, CH-8092 Zürich, Switzerland

sum of inner turning angles in an optimal solution always equals π. This implies that the problem can be solved by the standard ILP model without producing any integral subtours. Moreover, we give a simple constructive *polynomial time algorithm* to find such an optimal solution. If the number of vertices is even the optimal value lies between 0 and 2π and these two bounds are tight.